**09 – Linear Operators**

**Topics:** Linear differential operators/equations

**Summary:** Linear operators are defined, and students must determine which of five operators are linear. The second part addresses how the components of a complex solution are themselves solutions to a linear differential equation.

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**Comments:** Students should be able to finish these tasks in less than 15 minutes. They should be sure to check their answers to the first part, since many will mistakenly believe that option *IV* is linear if they don’t think too hard about it. The final task is designed to help students not get confused about how to translate between these complex exponential representations and physical solutions.

**A.** A *function*  takes a number  as input and outputs another number .

An *operator*  takes a function  as input and outputs another function . A *linear* operator is one where 

*A* & *B* are numbers, and  &  are functions.

Which of the following are linear operators? Check each of them to be sure.

I. 

II. 

III. 

IV. 

IV. 

**B.** Consider the differential equation: 

where *A*, *B* & *C* are constants.

Suppose we find complex solutions  and :



 and  each have real and imaginary parts:

 & 

Given the complex solutions, is it always true that  and  are also solutions? In other words, is the following always true?

 

Briefly explain your reasoning.